emitted from the cavity). The theory concerned with this specific solution (and therefore appropriate to describe laser light propagation) is called gaussian beam optics.

8.2.1 Gaussian Beam Properties

Figure 8.1 depicts a gaussian beam propagating along the z-axis. The intensity distribution is radially symmetric and of gaussian shape.

$$I(\rho) = \underbrace{P \cdot \frac{B}{\pi}}_{I_0} e^{-B\rho^2}, \quad B = \frac{2}{w^2}, \quad \rho = \sqrt{x^2 + y^2}, \quad P = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(\mathbf{r})\, dx\, dy \tag{8.1}$$

The beam radius $w(z)$ at a certain position z is usually defined as a drop in intensity to $1/e^2$. The radius at the narrowest point of the beam is called the waistsize w_0 or focus spotsize. Indicated by the dashed lines is the curvature of the wavefront.

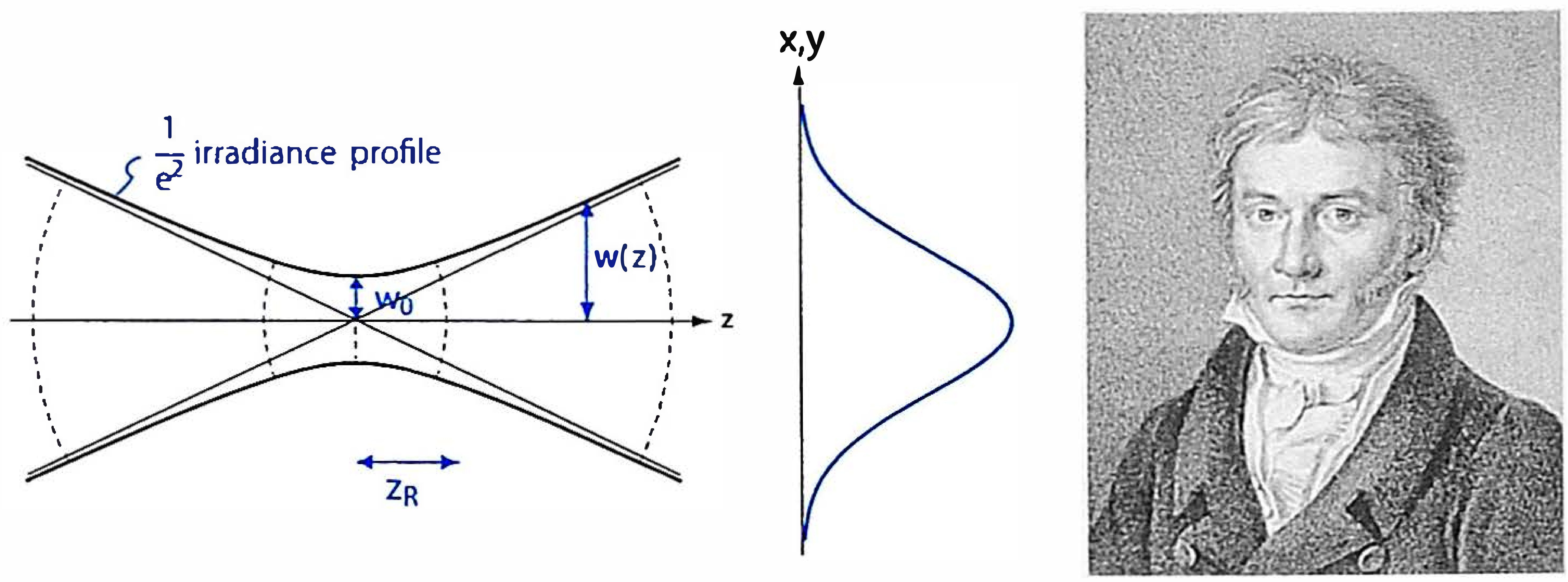

Figure 8.1 Gaussian spherical beam propagating in the z-direction, Carl Friedrich Gauss

Mathematically the beam radius $w(z)$ and radius of curvature $R(z)$ of the wavefronts are given by

$$w(z) = w_0 \sqrt{1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2} \rightarrow w(z) \approx \frac{\lambda z}{\pi w_0} \tag{8.2a}$$

$$R(z) = z \left[1 + \left(\frac{\pi w_0^2}{\lambda z}\right)^2\right]. \tag{8.2b}$$

The approximation given for the irradiance profile is valid for large distances z from the beam waist.

Two important things become evident from Eq. 8.2. First, the entire gaussian beam is determined by only two parameters, namely size and position of its waist. Second, the radius at a fixed position z is inversely proportional to the waist size. In other words: "The smaller the waist - the stronger the beam divergence."

The radius of curvature of the wavefront $R(z)$ equals infinity at the waist, decreases with increasing z, reaching a minimum of $2z_R$ at the Rayleigh range $z_R = \pi w_0^2/\lambda$ (indicating the distance where the cross sectional area of the beam has doubled compared to the waist), and increases again, asymptotically approaching z.

8.2.2 Gaussian Beam Propagation

If a gaussian beam is transmitted through a set of circularly symmetric optical components aligned with the beam axis, the gaussian beam remains a gaussian beam.

Often a strong positive lens is used to focus a reasonably well collimated gaussian beam to a very small spot. In this case it is valid to assume that the beam will be focused in the focal plane of the lens[1] (Fig 8.2a), which yields a simple relation between the diameter D of the incoming beam, focal length f of the lens and waist size of the outgoing beam w_0 by direct application of Eq. 8.2a:

$$2w_0 = \left(\frac{4\lambda}{\pi}\right)\left(\frac{f}{D}\right) \tag{8.3}$$

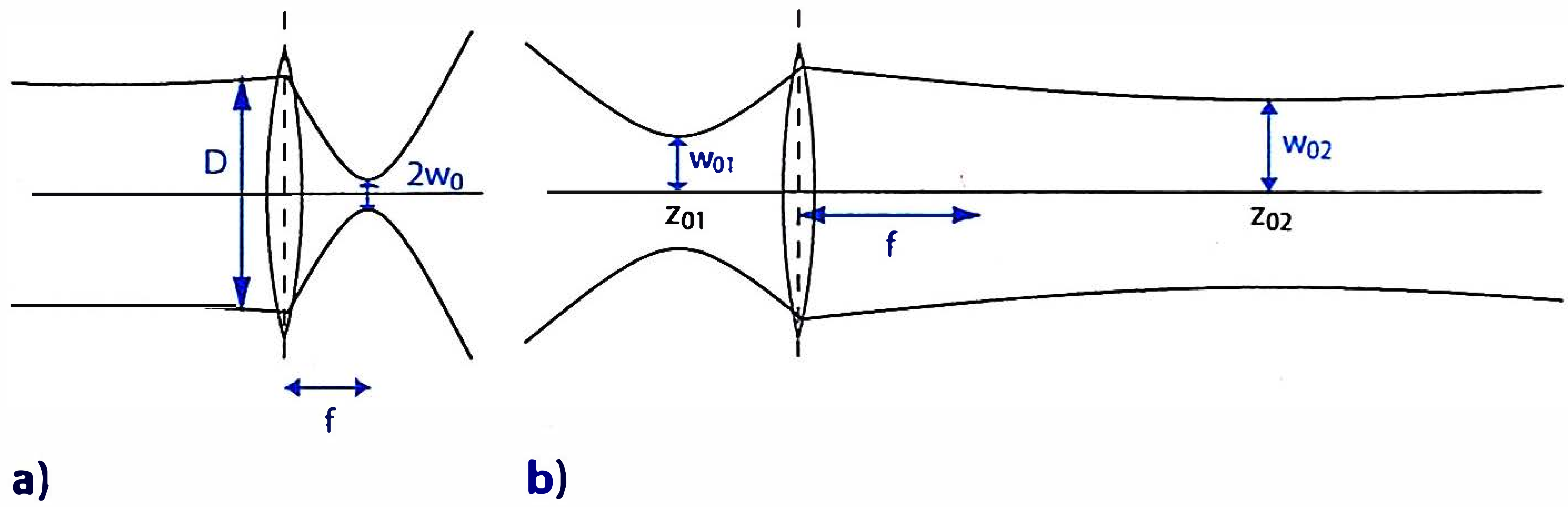

Figure 8.2 A well collimated gaussian beam is focused in the focal plane of a strong positive lens, properly described by Eq. 8.3 (a) whereas ABCD matrix formalism has to be used to describe the general case (b).

In general, transmission of a gaussian beam through an arbitrary paraxial optical system can be described by a formalism similar to ABCD (or ray transfer) matrix analysis known from geometrical optics. Incoming and outgoing beam at the boundaries of an optical element are represented by their complex beam parameter $q(z)$ defined as

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[1] As mentioned before, a gaussian beam has a maximum wavefront curvature-radius given by two times the Rayleigh range (which is proportional to the square of w_0). Therefore, when going through a strong positive lens (which itself induces a strong change in curvature of the wavefront) a beam with big enough w_0 can be treated like a plain wave which, by definition, gets focused in the focal plane of the lens.